L’Hospital’s Rule Can be Used to Evaluate $\lim_{x \to 0} \frac{\sin x}{x}$

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L’HOSPITAL’S RULE

Suppose f and g are differentiable on $(a; b)$ and $g^0(x) \neq 0$ for $a < x < b$. If $\lim_{x \to a^+} f(x) = \lim_{x \to a^+} g(x) = 0$ and $\lim_{x \to a^+} \frac{f^0(x)}{g^0(x)} = L$, then $\lim_{x \to a^+} \frac{f(x)}{g(x)} = \lim_{x \to a^+} \frac{f^0(x)}{g^0(x)} = L:$

Abstract

The standard proof presented in calculus courses concludes that $\frac{d}{dx} \sin x = \cos x$ using the limit $\lim_{x \to 0} \frac{\sin x}{x} = 1$. A natural question then becomes can you logically use L’Hospital’s Rule on this limit? The objective of this project is to name another method to find $\frac{d}{dx} \sin x$ without using the limit $\lim_{x \to 0} \frac{\sin x}{x}$. Once this proof is established, L’Hospital’s Rule can then be used on this limit without any logical uncertainties.

1. The Limit

$$\lim_{\to 0^+} \frac{\sin}{} = 1:$$

SOLUTION.

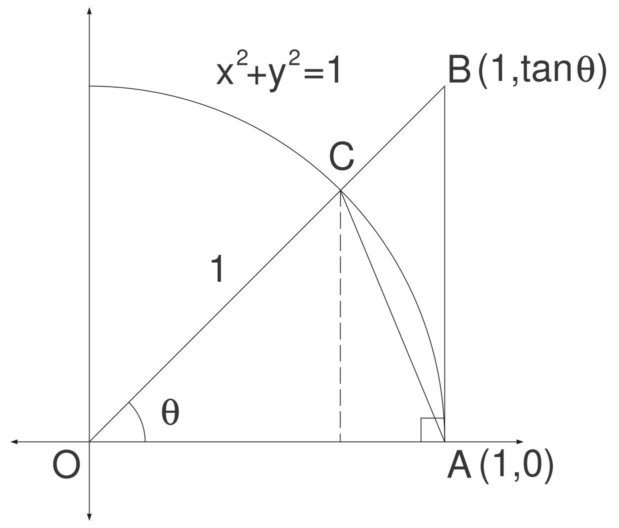

Figure 1

Using Figure 1, we obtain the following equations, which are valid for $0 < \quad < \quad =2:$

$$\text{area of triangle } OAC = \frac{1}{2} \text{ base } \text{ height} = \frac{1}{2} \; 1 \; \sin \quad = \frac{\sin}{}$$